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THE INHOUR FORMULA FOR A CIRCULATING-FUEL NUCLEAR REACTOR WITH SLUG FLOW

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As pointed out in a previous paper,[1] the circulating-fuel reactor differs in its dynamic behavior from a reactor with stationary fuel, because fuel circulation sweeps some of the delayed-neutrons precursors out of the reacting zone, and some delayed neutrons are given off in locations where they do not contribute to the chain reactions. One of the consequences of these circumstances is the fact that the inhour-formula usually derived for stationary-fuel reactors,[2] requires some modification before it becomes applicable to the circulating-fuel reactor. The inhour formula gives the relation between an excess multiplication factor, introduced into the reactor, and the time constant T of the resulting rise in reactor power. If the inhour formula is known, then the easily measured time constant can be used to determine the excess multiplication factor, a procedure frequently used in the quantitative evaluation of the various arrangements causing excess reactivity. Furthermore, the proper design of control rods and their drive mechanisms depends on the inhour formula.

Frequently, the experiments evaluating small excess multiplication factors are carried out at low reactor power, and the reactor power will then not cause an increase in the reactor temperature. This case will be considered here. In that case, the time dependence of the reactor power P can be described by the following equation:

$$dP/dt = (1/\tilde{\tau}) \left[(k_{ex}-\beta)P + \beta \int_0^\infty D(s)\, P(t-s)\, ds \right] \qquad (1)^*$$

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1 William Krasny Ergen, The Kinetics of the Circulating-Fuel Nuclear Reactor, J. Appl. Phys. (in print)

2 See for instance S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand Co., Inc., 1952, p. 294 ff.

* Some authors, for instance Glasstone and Edlund, loc. cit., write the equations corresponding to (1) in a slightly different form. The difference consists in terms of the order $k_{ex}(\tilde{\tau}/T)$ or $k_{ex}\beta$, which are negligibly small.

τ is the average lifetime of the prompt neutrons and k_{ex} the excess multiplication factor (or excess reactivity). The meaning of β and D(s) for a circulating-fuel reactor has been discussed in some detail in ref.[1] We approximate in the following the actual arrangement by a reactor for which the power distribution and the importance of a neutron are constant and for which all fuel elements have the same transit time $\theta_1 - \theta$ through the outside loop. In this approximation, β D(s) is simply the probability that a fission neutron, caused by a power burst at time zero, is a delayed neutron, given off inside the reactor at a time between s and s+ds. D(s) is normalized so that

$$\int_0^\infty D(s)\, ds = 1 \qquad (2)$$

If the fuel is stationary, $\beta D(s)$ is the familiar curve obtained by the superposition of 5 exponentials:

$$\beta D(s) = \sum_{i=1}^{5} \beta_i \lambda_i e^{-\lambda_i s} \qquad (3)$$

The λ_i are the decay constants of the 5 groups of delayed neutrons, and the β_i are the probabilities that a given fission neutron is a delayed neutron of the i^{th} group.

For the circulating-fuel reactor we first consider the fuel which was present in the reactor at time zero. At any time s, only a fraction of this fuel will be found in the reactor. This fraction is denoted by F(s), and by multiplying the right side of (3) by F(s), we obtain the function $\beta D(s)$ for the circulating-fuel reactor.

Since θ_1 is the total time required by the fuel to pass through a complete cycle, consisting of the reactor and the outside loop, it is clear that

at $s = n\Theta_1$ $(n = 0, 1, 2 \ldots)$, $F(s)$ is equal to 1;

at $s = n\Theta_1+\Theta$ $(n = 0, 1, 2 \ldots)$, $F(s)$ is equal to zero.

(We assume $\Theta_1 \geqq 2\Theta$ so that the fuel under consideration has not started to re-enter the reactor when the last of its elements leaves the reacting zone). Between $s = n\Theta_1$ and $s = n\Theta_1+\Theta$, $F(s)$ decreases linearly, and hence has the value $(n\Theta_1+\Theta-s)/\Theta$. At $s = n\Theta_1-\Theta (n= 1, 2, 3 \ldots)$, $F(s)$ is zero, but since the fuel under consideration re-enters the reactor between this moment and $s = n\Theta_1$, $F(s)$ increases linearly: $F(s) = (s-n\Theta_1+\Theta)/\Theta$. For $\beta D(s)$ we thus obtain:

$$\left.\begin{aligned} \beta D(s) &= \frac{n\Theta_1+\Theta-s}{\Theta}\sum_{i=1}^{5}\beta_i\lambda_i e^{-\lambda_i s} && \text{for } n\Theta_1 \leqq s \leqq n\Theta_1+\Theta, \quad n = 0, 1, 2, \ldots, \\ \beta D(s) &= \frac{s-n\Theta_1+\Theta}{\Theta}\sum_{i=1}^{5}\beta_i\lambda_i e^{-\lambda_i s} && \text{for } n\Theta_1-\Theta \leqq s \leqq n\Theta_1, \quad n = 1, 2, 3, \ldots, \\ \beta D(s) &= 0 && \text{for } n\Theta_1+\Theta \leqq s \leqq (n+1)\Theta_1-\Theta, \quad n = 0, 1, 2, \ldots \end{aligned}\right\} \quad (4)$$

Eq. (4) is now substituted into eq. (1), and for P we set $P = P_o e^{t/T}$. Then

$$\frac{\tau}{T} P_o e^{t/T} = (k_{ex}-\beta)P_o e^{t/T} + \sum_{i=1}^{5}\beta_i\lambda_i \left[\sum_{n=0}^{\infty}\int_{n\Theta_1}^{n\Theta_1+\Theta} \frac{n\Theta_1+\Theta-s}{\Theta} e^{-\lambda_i s} P_o e^{(t-s)/T}\,ds \right.$$

$$\left. + \sum_{n=1}^{\infty}\int_{n\Theta_1-\Theta}^{n\Theta_1} \frac{s-n\Theta_1+\Theta}{\Theta} e^{-\lambda_i s} P_o e^{(t-s)/T}\,ds \right]$$

The common factor $P_o e^{t/T}$ cancels out. The substitution $\sigma = n\Theta_1+\Theta-s$ transforms

$$\int_{n\theta_1}^{n\theta_1+\theta} (n\theta_1+\theta-s)\exp\left\{-\left[\lambda_i+(1/T)\right] s\right\} ds$$

into

$$\exp\left\{-n\theta_1\left[\lambda_i+(1/T)\right]\right\} \quad \exp\left\{-\theta\left[\lambda_i+(1/T)\right]\right\} \int_0^{\theta} \sigma \left\{\exp\left[\lambda_i+(1/T)\right]\sigma\right\} d\sigma$$

and the substitution $\sigma = s-n\theta_1+\theta$ transforms

$$\int_{n\theta_1-\theta}^{n\theta_1} (s-n\theta_1+\theta) \exp\left\{-\left[\lambda_i+(1/T)\right] s\right\} ds$$

into

$$\exp\left\{-n\theta_1\left[\lambda_i+(1/T)\right]\right\} \quad \exp\left\{\theta\left[\lambda_i+(1/T)\right]\right\} \int_0^{\theta} \sigma \exp\left\{-\left[\lambda_i+(1/T)\right]\sigma\right\} d\sigma .$$

The geometric series $\exp\left\{-n\left[\lambda_i+(1/T)\right]\theta_1\right\}$ can now be summed, and the integrals over σ evaluated by elementary methods. After performing all these operations, one obtains the following inhour formula:

$$k_{ex} = \frac{\tilde{\ell}}{T} + \beta$$

$$-\frac{1}{\theta}\sum_{i=1}^{5}\beta_i\lambda_i \frac{\mu_i\theta-1+e^{-\mu_i\theta}-e^{-\mu_i\theta_1}(\mu_i\theta+1)+e^{-\mu_i(\theta_1-\theta)}}{\mu_i^{2}\left[1-e^{-\mu_i\theta_1}\right]}, \qquad (5)$$

$$\mu_i = \lambda_i + (1/T). \qquad (6)$$

β is evaluated by means of eq. (2):

$$\beta = \int_0^{\infty} \beta D(s)\, ds.$$

If $\beta D(s)$ is substituted into this integral, expressions are obtained which are of the same type as the ones just discussed, and which can be evaluated by the same methods. The result is

$$\beta = \sum_{i=1}^{5} \beta_i \frac{\lambda_i\theta - 1 + e^{-\lambda_i\theta} - e^{-\lambda_i\theta_1}(\lambda_i\theta + 1) + e^{-\lambda_i(\theta_1-\theta)}}{\theta\lambda_i(1 - e^{-\lambda_i\theta_1})} \qquad (7)$$

In spite of the formidable appearance of eqs. (5), (6), and (7), it is easy to find, for any given θ and θ_1, the value of k_{ex} which produces a given time constant T.

Furthermore, the following reasoning describes the general features of the equations. Consider first the dependence of k_{ex} on T. If $T = \infty$, $\mu_i = \lambda_i$, and the sum on the right of (5) is equal to β, k_{ex} is equal to zero. This corresponds to the state in which the reactor is just critical. If T becomes very small, the μ_i become very large and in the fraction on the right of (5) the numerator is dominated by $\mu_i\theta$ and the bracket in the denominator by 1. Hence the fraction tends to zero like θ/μ_i, as T goes to zero. If τ is very small, as it is in practice, T will be small as soon as k_{ex} exceeds β by a small amount. Then the complicated sum on the right of (5) is of little importance, and T is determined by $\tau/T = k_{ex} - \beta$, that is the reactor period is inversely proportional to the excess of the reactivity over the reactivity corresponding to the "prompt critical" condition.

In the stationary-fuel reactor, the k_{ex} which make the reactor prompt critical is given by $\sum\beta_i$. With circulating fuel, the reactor is prompt critical if $k_{ex} = \beta$, which is less than $\sum\beta_i$, that is it takes less excess reactivity to make the circulating-fuel reactor prompt critical than to do the same thing to a stationary-fuel reactor. This is physically evident because the fuel circulation renders some of the delayed neutrons ineffective. That $\beta < \sum\beta_i$ can also be verified mathematically.

For T intermediate between very small positive values and $+\infty$ we consider again the analogy to the stationary fuel reactor. Here the inhour formula reads:

$$k_{ex} = \frac{\tilde{l}}{T} + \sum_{i=1}^{5} \frac{\beta_i}{1 + \lambda_i T} \qquad *) \qquad (8)$$

For every positive k_{ex} there is one, and only one, positive time constant T and vice versa. This in a consequence of the fact that k_{ex} is a monoton decreasing function of T, for if this monotony did not exist, there could be several positive T value corresponding to a given value of k_{ex}, or vice versa. In the circulating-fuel reactor the situation is qualitatively the same; the fractions under the sum in eq. (5) are essentially of the form

$$\frac{x - 1 + e^{-x} - e^{-ax}(x+1) + e^{-(a-1)x}}{x^2\left[1-e^{-ax}\right]} \qquad (x = \mu_i \theta \quad a = \theta_1/\theta)$$

and an expression of this form can be shown to be a monoton decreasing function of positive x [2]; the expression is thus a monoton increasing function of T (see eq. (6)), and as T increases k_{ex} decreases monotonically, according to eq. (5); for every positive k_{ex} there is one, and only one, positive T, and vice versa. This, of course, does not preclude that there exist for a given k_{ex} several negative T values, in addition to the one positive value. Negative T correspond, however, to decaying exponentials, which are of no importance if the rise in power is observed for a sufficiently long time.

Consider now the behavior of eq. (5) with variation of θ, the transit time of the fuel through the reactor, and of θ_1, the transit time of the fuel through the whole loop. If θ (and hence also θ_1) is very large compared to all $1/\lambda_i$ and $1/\mu_i$, eq. (5) reduces to eq. (8), the inhour formula for the stationary fuel reactor. The circulation is so slow that the reactor behaves as if the fuel were

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*) See Glasstone and Edlund, loc. cit. p. 301, eq. 10.29.1. See also preceding footnote.

2 W. K. Ergen, The Behavior of Certain Functions Related to the Inhour Formula of Circulating Fuel Reactors, Oak Ridge National Laboratory Memo CF 54-1-1 Jan. 15, 1954.

stationary, inasmuch as all delayed neutrons, even the ones with the long-lived precursors, are given off inside the reacting zone, before much fuel reaches the outside. On the other hand, if Θ_1 and hence also Θ is small compared to all $1/\lambda_i$, that is if the transit time of the fuel through the complete loop is small compared to the mean life of even the short-lived delayed-neutrons precursors, then for $T \gg \Theta$

$$k_{ex} = \frac{\tilde{\tau}}{T} + \frac{\Theta}{\Theta_1} \sum_{i=1}^{5} \frac{\beta_i}{1 + \lambda_i T}. \qquad (9)$$

This is the same as the inhour formula for the stationary fuel reactor, except that all the fission yields β_i are decreased by the factor Θ/Θ_1. This is physically easy to understand, since Θ/Θ, is just the probability that a given delayed neutron is born inside the reactor.

Of interest is the intermediate case, in which Θ is smaller than the mean life of the long-lived delayed-neutron precursors, and larger than the mean life of the short-lived precursors. In that case, the long-delayed neutrons act approximately according to eq. (9) and are reduced by the factor Θ/Θ_1. On the other hand, the neutrons with the short-lived precursors behave approximately like in eq. (8) and are not appreciably reduced. Hence, a small excess reactivity enables the reactor to increase its power without "waiting" for the not very abundant long-delayed neutron. The reactor goes to fairly short time constants with surprisingly small excess reactivities. However, to make the reactor prompt critical, that is to enable it to exponentiate without even the little-delayed neutrons, takes a substantial excess reactivity because of the almost undiminished amount of the latter neutrons.